2.1 Product and Quotient Rules: Form B Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_form.py`). All problems are original; provenance is in `PROVENANCE_FORMS.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Show work.** A bare correct answer earns at most half credit, except where a row says otherwise.
- **Any valid method.** Simplifying first, the product rule, and the quotient rule are all accepted wherever they apply; no method is required, and long but correct work gets full credit.
- **Value vs. derivative.** A function value where a derivative was asked (or the reverse) earns 0 for that part unless a row says otherwise.
- **Domains.** On Q1, a correct note on where f' does not exist is accepted, never required.
- **Simplifying.** Answers need not be simplified beyond what a row says; equivalent forms are correct.

Form B

Item	Answer	Rubric / trap
1(a)	$\dfrac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$	product rule: $\left(\frac{1}{2}x^{-1/2}\right)(\sin x) + \sqrt{x}\,(\cos x)$. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* multiplying the derivatives is not a rule; earns 0.
1(b)	$9x^2 + 2x - 6$	product rule $3(x^2-2) + (3x+1)(2x)$, or expand to $3x^3 + x^2 - 6x - 2$ first. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* $(3)(2x)$, the product of the derivatives, earns 0.
1(c)	$\dfrac{-x^2 - 4x + 5}{(x^2+5)^2}$	quotient rule: $\dfrac{1\cdot(x^2+5) - (x+2)(2x)}{(x^2+5)^2}$. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* $-(x+2)(2x) = -2x^2 - 4x$; reversing the numerator flips every sign.
1(d)	$3 + x^{-2}$	simplify first: $3x - x^{-1}$. The quotient rule also works. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* $-x^{-1}$ has derivative $+x^{-2}$; dividing only the first term by $2x^3$.
1(e)	0 (for every x in the domain of f)	$\dfrac{2x^2(x-3)}{x^2(x-3)} = 2$ wherever f is defined, a constant. The quotient rule, carried through, also gives 0. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* a constant in disguise; the quotient rule is valid but long.
1(f)	$-4\csc x \cot x$	$\sin x \csc x = 1$ wherever $\csc x$ is defined, so $f(x) = 4\csc x + 1$. The product rule on $\sin x \csc x$ also gives 0. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* a constant hidden in a product; the derivative of $\csc x$ is $-\csc x \cot x$, with a minus sign.
1(g)	$2x\cot x - x^2\csc^2 x$	product rule: $(2x)(\cot x) + (x^2)(-\csc^2 x)$. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* the minus sign in the derivative of $\cot x$, which is $-\csc^2 x$.
1(h)	0	$\sec\left(\frac{\pi}{3}\right) = 2$ and $\cos(\pi) = -1$ are numbers, so $f(x) = -2$ for every x. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* no x appears; the product rule applied as if these were functions of x earns 0.

Item	Answer	Rubric / trap
2	$-\frac{2}{3}$	2 pts $f'(x) = \dfrac{-\sin x(1+\sin x) - \cos x \cos x}{(1+\sin x)^2} = \dfrac{-1}{1+\sin x}$; 2 pts substitution; 1 pt value. *Trap:* $\sin^2 x + \cos^2 x = 1$ collapses the numerator; reversing it gives $+\frac{2}{3}$.
3(a)	$2\sqrt{3}$	the h-form of $f'\left(\frac{\pi}{3}\right)$ for $f(x) = \sec x$, since $\sec\frac{\pi}{3} = 2$; $\sec\frac{\pi}{3}\tan\frac{\pi}{3} = 2\sqrt{3}$. 1 pt naming f and c (or any valid evaluation), 2 pts value. *Trap:* the 2 is $\sec\frac{\pi}{3}$ in disguise; plugging in $h = 0$ gives $\frac{0}{0}$.
3(b)	-1	the c-form of $f'\left(\frac{\pi}{2}\right)$ for $f(x) = \cot x$, since $\cot\frac{\pi}{2} = 0$; $-\csc^2\frac{\pi}{2} = -1$. 1 pt naming f and c (or any valid evaluation), 2 pts value. *Trap:* the missing $-f(c)$ is $-\cot\frac{\pi}{2} = 0$.
4	$y - 3 = -(x-3)$, i.e. $y = -x + 6$	1 pt $f(3) = 3$; 2 pts $f'(x) = \dfrac{(x-1)-(x+3)}{(x-1)^2} = \dfrac{-4}{(x-1)^2}$; 1 pt $f'(3) = -1$; 2 pts equation, any correct form. *Trap:* reversed numerator gives slope $+1$.
5	2	2 pts $f(2) = 3$, so the tangent passes through $(2,3)$; 3 pts slope through $(2,3)$ and $(-1,-3)$: $\dfrac{3-(-3)}{2-(-1)} = 2$. *Trap:* no rule in this course differentiates $\sqrt{x^3+1}$, and none is needed: two points on the tangent line give its slope, which is $f'(3)$.
6	$x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$	2 pts $f'(x) = 4 - \csc^2 x$; 2 pts $\sin^2 x = \frac{1}{4}$, $\sin x = \pm\frac{1}{2}$; 2 pts both solutions in the interval and nothing else. *Trap:* $\sin x = \frac{1}{2}$ has two solutions in $(0,\pi)$; stopping at $\frac{\pi}{6}$ costs 2.
7	$-\csc^2 x$ (shown)	1 pt rewrite $\cot x = \dfrac{\cos x}{\sin x}$; 2 pts quotient rule $\dfrac{(-\sin x)\sin x - \cos x \cos x}{\sin^2 x}$; 2 pts $= \dfrac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \dfrac{-1}{\sin^2 x}$; 1 pt $= -\csc^2 x$. *Trap:* quoting the cot rule earns 0; the numerator is $-(\sin^2 x + \cos^2 x)$, not $\sin^2 x - \cos^2 x$.
8(a)	$\frac{7}{4}$	$h'(1) = \dfrac{g'(1)f(1) - g(1)f'(1)}{[f(1)]^2}$, with the row $x = 1$. 1 pt rule, 2 pts entries, 1 pt value. *Trap:* h has g on top; the f/g version gives $-\frac{7}{9}$; reversed numerator gives $-\frac{7}{4}$. Columns run x, $f(x)$, $g'(x)$, $f'(x)$, $g(x)$.
8(b)	14	$k'(3) = f'(3)g(3) + f(3)g'(3)$, with the row $x = 3$. 1 pt rule, 2 pts entries, 1 pt value. *Trap:* $f'(3)g'(3) = -8$; $f(3)g(3) = -3$.
8(c)	2	$m'(x) = \dfrac{g'(x)}{f(1)}$ because $f(1) = 2$ is a number, so $m'(-2) = \dfrac{g'(-2)}{f(1)}$, with the row $x = -2$. 2 pts recognizing the constant denominator, 2 pts value. *Trap:* $f(1)$ is a number; the quotient rule with $f'(1) = 1$ gives $\frac{3}{2}$; earns at most 1.
9(a)	3	$h'(x) = f'(x)g(x) + f(x)g'(x)$, $7 = 3f'(-1) + (-2)(1)$; substitute the given values and solve for $f'(-1)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* the item gives the derivative of the product and asks for a factor's derivative; $h' = f'g'$ gives 7.
9(b)	5	$k'(x) = \dfrac{2x\,g(x) - x^2 g'(x)}{[g(x)]^2}$, $-\frac{1}{4} = \dfrac{16 - 4g'(2)}{16}$; substitute the given values and solve for $g'(2)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* reversing the numerator gives 3.
9(c)	6	$w'(x) = \dfrac{x\,f'(x) - f(x)}{x^2}$, $1 = \dfrac{2(5) - f(2)}{4}$; substitute the given values and solve for $f(2)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* this part asks for a function value, $f(2)$; reversing the numerator gives 14.
10(a)	-5	Readings: $f(-3) = 1$, $f'(-3) = -2$, $g(-3) = -2$, $g'(-3) = -1$ (slopes are rise over run of the segment containing $x = -3$). $q'(-3) = \dfrac{g'(-3)f(-3) - g(-3)f'(-3)}{[f(-3)]^2}$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* q has g on top: the f/g version gives $\frac{5}{4}$; reversed numerator gives 5; reading a height where a slope is needed.

Item	Answer	Rubric / trap
10(b)	4	Readings: $f(3) = 2$, $f'(3) = -1$, $g(3) = -2$, $g'(3) = 1$ (slopes are rise over run of the segment containing $x = 3$). $p'(3) = f'(3)g(3) + f(3)g'(3)$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* $f'(3)g'(3) = -1$; $f(3)g(3) = -4$; mixing a value from one graph with a slope from the other.
10(c)	1	Readings: $f(1) = 1$, $f'(1) = 2$, $g(1) = -1$, $g'(1) = -2$ (slopes are rise over run of the segment containing $x = 1$). $r'(1) = \dfrac{1 \cdot f'(1) - f(1) \cdot 1}{1^2}$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* reversed numerator gives -1; $f'(1)/1 = 2$ ignores the quotient rule.
11	IV	6 pts, all or nothing. In IV the dashed curve is $f(x) = \frac{x^3}{2} - \frac{3x^2}{2}$ and the solid curve is $f'(x) = \frac{3x^2}{2} - 3x$: it is zero exactly where the dashed curve turns ($x = 0, 2$), positive where that curve rises, and negative where it falls. Wrong choices: I: the line is the derivative of the derivative: it is zero at $x = 1$, not where the cubic turns; II: right zeros, wrong sign: it is positive where the cubic falls (between $x = 0$ and $x = 2$); III: shifted: the other curve is zero at $x = 1$ and $x = 3$, not at the turning points $x = 0$ and $x = 2$. *Trap:* assuming the dashed curve is always the derivative; the styles swap between graphs.

Points: $24 + 5 + 6 + 6 + 5 + 6 + 6 + 12 + 12 + 12 + 6 = 100$.